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Evolutionary Behaviour of Newly Constructed Accelerating Universe in the Framework of Teleparallel Gravity

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Abstract: In this study, we take into account a particular form of $f(T)$ inspired by a $f(R)$ gravity model. The Monte Carlo Markov Chain (MCMC) approach is used to constrain the parameters of the Hubble model, offering a strong statistical analysis. Our outcomes confirm the viability of the suggested $f(T)$ model as an alternative framework for dark energy explanation by successfully supporting an accelerating cosmic expansion.

Keywords: Teleparallel Gravity, Monte Carlo Markov Chain, Energy Conditions

Introduction. Our understanding of cosmology has been drastically transformed by observational findings made in 1998 [1] that the universe is expanding in an accelerated manner. Due to several drawbacks of GR, Hence, the establishment of modified theories of gravity is one of the main approaches being explored to study the late time cosmic evolution.

In an Intriguing extension of Riemannian geometry, in the witzenböck space, the Riemannian curvature tensor is zero, and the torsion tensor is non-zero. This leads to a flat spacetime with the significant characteristic of absolute parallelism, also referred to as teleparallelism. The primary characteristic of the teleparallel approach is the substitution of a set of tetrad vectors, for the metric tensor, which is the main physical variable that determines gravitational qualities. Instead of curvature, torsion, originating from the tetrad fields, can be used as a general description of the gravitational effect. This is reflected to the theory known as the teleparallel equivalent of general relativity (TEGR), which has been extended to the $f(T)$ gravity theory.

The primary objective of this research is to explore the $f(T)$ modified gravity, starting with TEGR rather than the GR. In this study, we employ the simple parametrization of a particular form of the Hubble parameter using Hubble, BAO, and Hubble+BAO observational datasets and generate exact solutions for the modified Friedmann equations in the FLRW spacetime. Specifically we employ a form of

$f(T)$, $f(T) = T + \frac{\alpha t(T^2/T_0^2)^n}{1+(T^2/T_0^2)^n}$, where α and n are constants with $T = -6H_0^2$, where

H_0 is the current value of the Hubble parameter. Subsequently, we have examined all the dynamical and cosmological parameters followed by energy conditions and $Om(z)$ diagnostic. The work is organized as follows: in section 2, we commence by offering an overview to TG and then describe the $f(T)$ gravity framework.

In section 3, a specific form of the Hubble parameter is parametrized using Hubble, BAO and Hubble+BAO datasets via MCMC approach. Section 4 presents solutions to the field equations and explores the cosmological elements of the specific form of $f(T)$. In sections 5 and 6, the viability of the model is checked by performing energy conditions and $Om(z)$ diagnostic respectively. Section 7 provides a thorough conclusion to the work.

Methods and materials. Framework of $f(T)$. The torsion scalar-based modified theory of gravity, known as $f(T)$, has a geometric action determined by an algebraic function associated with torsion. To describe the geometric components, this theory incorporates orthonormal tetrad components that are defined within the tangent space at each point of the manifold. Basically, the line element is presented as:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ij} \theta^i \theta^j$$

where $dx^\mu = e_i^\mu \theta^i$, $\theta^i = e_\mu^i dx^\mu$ and $\eta_{ij} = \text{diag}(1, -1, -1, -1)$ represents the metric associated with Minkowski spacetime and e_μ^i represents the components of the tetrad satisfying the conditions:

$$e_\mu^i e_j^\mu = \delta_j^i$$

The action [2] of the $f(T)$ gravity can be defined as,

$$S = \frac{1}{16\pi G} \int d^4x e [T + f(T) + \mathcal{L}_m].$$

where $\mathcal{L}_m$ is the total matter Lagrangian, $e = \det[e_\mu^i] = \sqrt{-g}$, and G is the gravitational constant. The natural system, $\kappa^2 = 8\pi G = 1$, is taken into consideration. The gravitational field equations can be found by varying the action with respect to vierbein as,

$$e^{-1} \partial_\mu (e e_a^\rho S_\rho^{\mu\nu}) [1 + f_T] + e_a^\rho S_\rho^{\mu\nu} \partial_\mu (T) f_{TT} - e_a^\lambda T_{\mu\lambda}^\rho S_\rho^{\mu\nu} [1 + f_T] + \frac{1}{4} e_a^\nu [T + f(T)] = 4\pi G e_a^\rho T_\rho^\nu$$

T_ρ^ν is the total-energy momentum tensor, and we represent $f = f(T)$ and the first and second derivatives of f with regard to T as f_T and f_{TT} , respectively. In this study, we will consider the FLRW metric to derive the modified Friedman equations. Then the $f(T)$ gravity field equations are:

$$3H^2 = 8\pi G (\rho_m + \rho_r) - \frac{f}{2} + T f_T;$$

$$\dot{H} = - \frac{4\pi G (\rho_m + \rho_r + p_m + p_r)}{1 + f_T + 2T f_{TT}}.$$

Where $H \equiv \frac{\dot{a}}{a}$ and the derivative of it with respect to cosmic time is represented by $\dot{H}$. It is now possible to describe the field equations of $f(T)$ gravity in the DE sector pressure (p_{de}) and energy density (ρ_{de}) as,

$$\rho_{de} \equiv \frac{1}{16\pi G} [-f + 2Tf_T]$$

and

$$p_{de} \equiv -\frac{1}{16\pi G} \left[\frac{-f + Tf_T - 2T^2f_{TT}}{1 + f_T + 2Tf_{TT}} \right].$$

Then the EoS parameter can be expressed as,

$$\omega_{de} \equiv -1 + \frac{(f_T + 2Tf_{TT})(-f + T + 2Tf_T)}{(1 + f_T + 2Tf_{TT})(-f + 2Tf_T)}.$$

Model Parametrization. From [3], we introduce the following new parametrization of the Hubble parameter in the FLRW universe:

$$H(z) = \sqrt{(z - a)(1 + bz) + (1 + a)},$$

where H_0 is the current value of the Hubble parameter at $z = 0$, a and b are free parameters that must be ascertained via observational constraints.

Analysis With Observational Datasets. In this part, we use observational data from several cosmological surveys to constrain the free parameters of $H(z)$, that is by investigating data describing the distance-redshift relation.

Hubble Data: A sequence of 32 data points for the Hubble parameter are taken in the redshift range $0.07 \leq z \leq 1.965$. The chi-square function for the Hubble dataset is defined as,

$$\chi_H^2(\psi) = \sum_{\{i=1\}}^{\{32\}} \frac{(H_{th}(z_i, \psi) - H_{obs}(z_i))^2}{\sigma_H^2(z_i)},$$

where ψ is the vector of the cosmological background parameters and $H_{th}(z_i, \psi)$ and $H_{obs}(z_i)$ refer to the developed and the observed Hubble parameters respectively. The observational errors in the observed values $H_{obs}(z_i)$ are denoted by $\sigma_H^2(z_i)$.

BAO Data: In this study, we utilize a dataset of 26 independent data points derived from line-of-sight BAO measurements. The χ^2 function for the BAO data set is defined by,

$$\chi_{BAO}^2(\phi) = \sum_{\{i=1\}}^{\{26\}} \frac{(H_{th}(z_i, \phi) - H_{obs}(z_i))^2}{\sigma_H^2(z_i)},$$

where $H_{th}(z_i, \phi)$ represents the theoretical value of the hubble, the model parameters are represented by ϕ , $H_{obs}(z_i)$ represents the observed value of the Hubble from the BAO analysis, and the related error in the observed BAO data points is denoted by $\sigma_H(z_i)$.

Once, Hubble and BAO datasets have been analyzed, we may also take the combination of both to determine the best-fit values of our model. To do this, the χ^2_{total} function can be set up as the sum of $\chi^2_{H(z)}$ and χ^2_{BAO} .

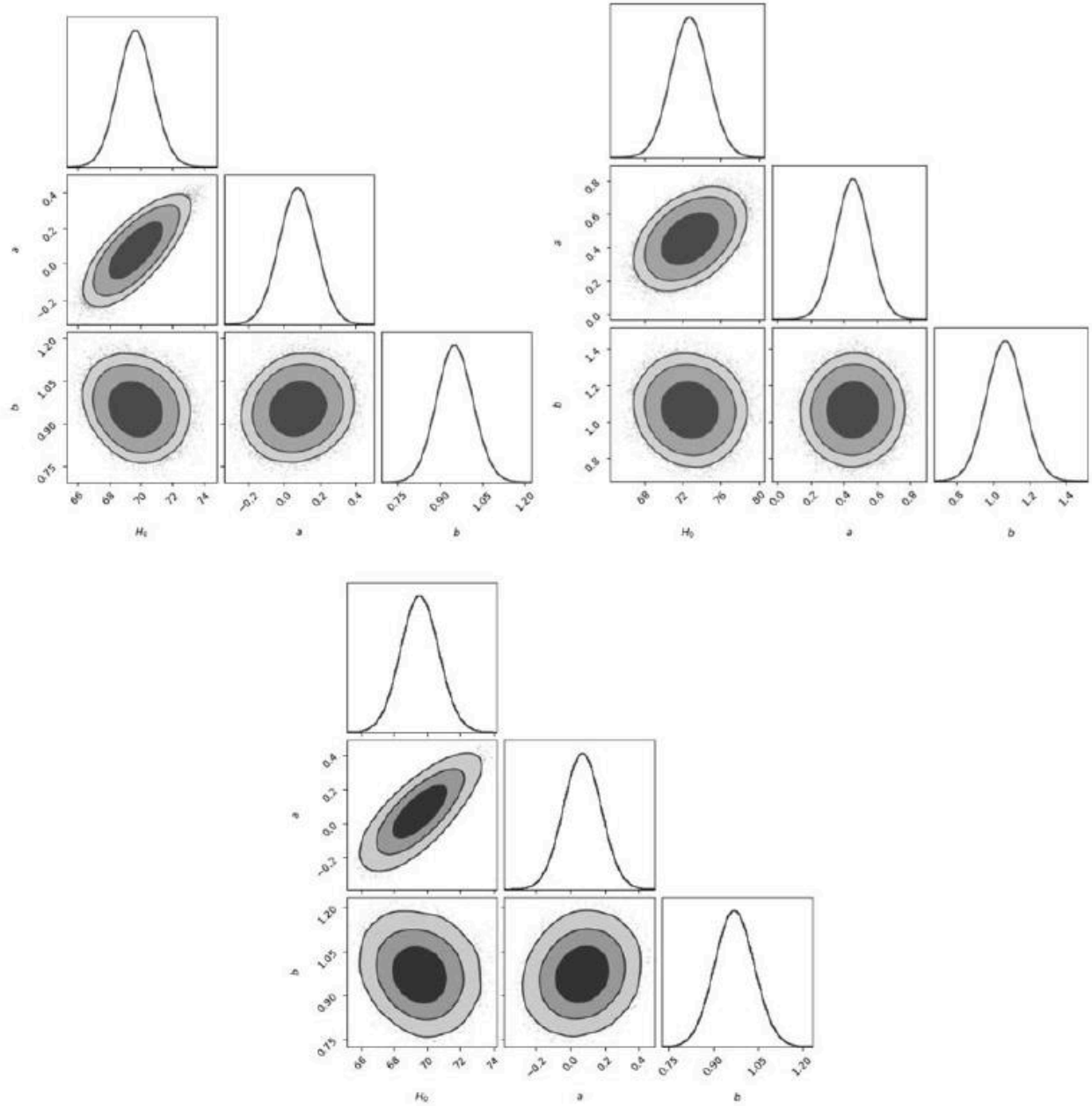


Fig. 1. 2D-contour plot derived from the study of Hubble, BAO and Hubble+BAO datasets in the (i)upper left panel, (ii)upper right panel, and (iii) lower panel that display the most likely values and confidence regions up to 3σ for model parameters a , b , and H_0

Table 1

Best-fit values with 1σ , 2σ , and 3σ confidence intervals for the model parameters a , b , and H_0 derived from the Hubble, BAO, and Hubble+BAO datasets

Datasets		a	b	H_0
Hubble	1σ	$0.45^{+0.10}_{-0.09}$	$1.07^{+0.09}_{-0.10}$	$72.76^{+1.83}_{-1.8}$
	2σ	$0.45^{+0.19}_{-0.18}$	$1.07^{+0.18}_{-0.19}$	$72.76^{+3.61}_{-3.53}$
	3σ	$0.45^{+0.29}_{-0.28}$	$1.07^{+0.28}_{-0.29}$	$72.76^{+5.43}_{-5.35}$
BAO	1σ	$0.076^{+0.098}_{-0.097}$	$0.952^{+0.060}_{-0.059}$	$69.683^{+1.035}_{-1.020}$
	2σ	$0.076^{+0.193}_{-0.192}$	$0.952^{+0.119}_{-0.115}$	$69.683^{+2.074}_{-1.998}$
	3σ	$0.076^{+0.288}_{-0.291}$	$0.952^{+0.182}_{-0.171}$	$69.683^{+3.197}_{-3.010}$
Hubble+BAO	1σ	$0.069^{+0.095}_{-0.096}$	$0.971^{+0.060}_{-0.058}$	$69.550^{+1.004}_{-0.998}$
	2σ	$0.069^{+0.188}_{-0.189}$	$0.971^{+0.117}_{-0.114}$	$69.550^{+2.004}_{-1.951}$
	3σ	$0.069^{+0.281}_{-0.286}$	$0.971^{+0.179}_{-0.171}$	$69.550^{+3.041}_{-2.959}$

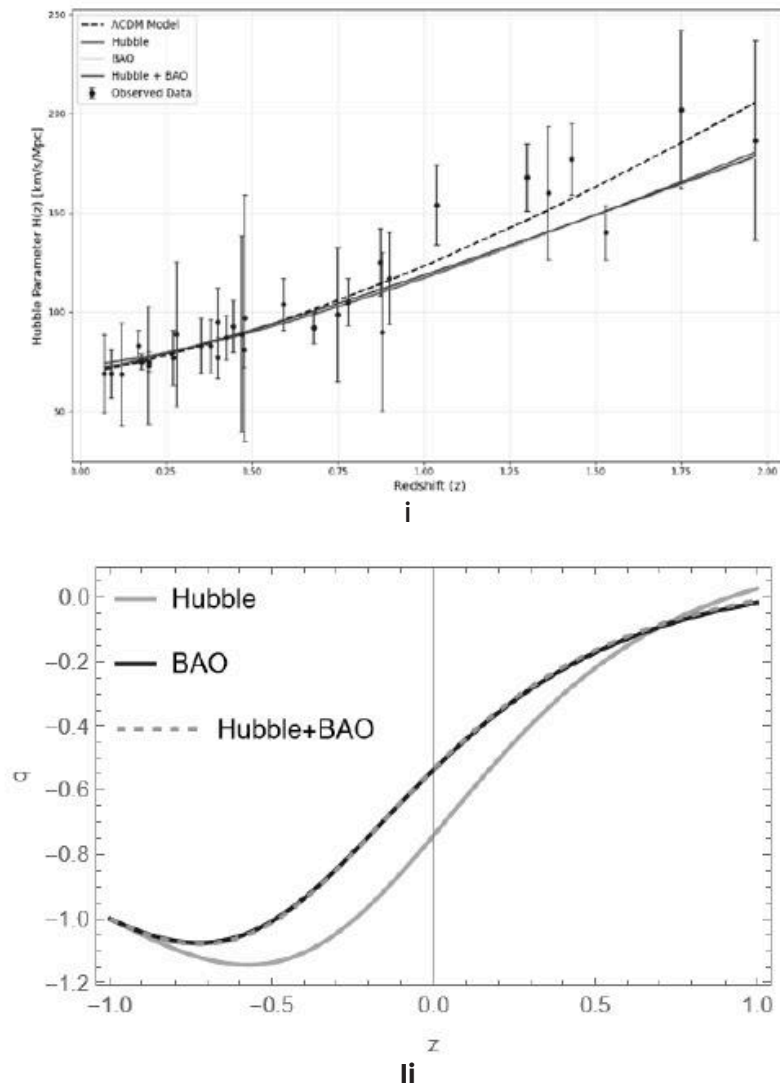


Fig. 2. (i) Left panel is the error bar plot of Hubble parameter, and (ii) right panel is the plot of deceleration parameter for Hubble, BAO and Hubble+BAO datasets

We can see that the error bar plot of the HP presented in Fig. 2 left Pannel is perfectly fit with the observational data and consistent with Λ CDM model. In Fig. 2 right Pannel, the behaviour of the deceleration parameter is staying negative throughout indicating that the expansion is slow at early time but increases when it moves towards late. Additionally, the estimation of the current values of the deceleration parameter for Hubble, BAO and Hubble+BAO datasets are $q_0 = -0.74$, $q_0 = -0.54$ and $q_0 = -0.53$ respectively which are in agreement with some previous literatures such as $q_0 = -0.584$ [4], $q_0 = -0.544$ [5], $q_0 = -0.55$ [6], $q_0 = -0.749$ [7], $q_0 = -0.744$ [8] etc.

Dynamical study with the model. In analogous to $f(R)$ theory, the form of $f(T)$ will be given as, $f(T) = T + \frac{\alpha t(T^2/T_0^2)^n}{1+(T^2/T_0^2)^n}$, [9], where α , n are constants, and $T = -6H_0^2$ with the first and second derivatives denoted as f_T and f_{TT} and defined as $\frac{2\alpha n T_0 (T^2/T_0^2)^n}{T((T^2/T_0^2)^n + 1)^2} + 1$ and, $-\frac{2\alpha n T_0 (T^2/T_0^2)^n ((2n+1)(T^2/T_0^2)^n - 2n+1)}{T^2((T^2/T_0^2)^n + 1)^3}$ respectively. Then we can reduce the field equations as,

$$\begin{aligned} \rho_{de} &= \frac{1}{2} \left[-6\zeta H_0^2 - \alpha T_0 \left(1 - \frac{36^n (4n+1)\tau+1}{(36^n \tau+1)^2} \right) \right]; \\ p_{de} &= -\frac{\alpha H_0^2 \zeta 2^{2n-1} 3^{2n+1} T_0 \tau (8n^2 + 1296^n \tau^2 - 2^{2n+1} 9^n (n+1)(4n-1)\tau - 6n+1)}{\alpha 36^n n T_0 \tau (-36^n (4n+1)\tau + 4n-1) - 6H_0^2 \zeta (36^n \tau+1)^3}, \\ \omega &= \\ &= -\frac{\alpha H_0^2 \zeta 2^{2n} 3^{2n+1} T_0 \tau (8n^2 + 1296^n \tau^2 + 2^{2n+1} 9^n (n+1)(4n-1)\tau - 6n+1)}{(\alpha 36^n n T_0 \tau (-36^n (4n+1)\tau + 4n-1) - 6H_0^2 \zeta (36^n \tau+1)^3) \left(-6\zeta H_0^2 - \alpha T_0 \left(1 - \frac{36^n (4n+1)\tau+1}{(36^n \tau+1)^2} \right) \right)}, \end{aligned}$$

Where $\zeta = (-abz + bz^2 + z + 1)$ and $\tau = \left(\frac{H_0^4 \zeta^2}{T_0^2} \right)^n$.

These are the parameters a , b , α , and n on which the behaviour of the energy density and the EoS parameter depend from which two parameters a and b are constrained by the MCMC approach and the remaining two, that is, α and n are selected in such a way as to obtain a positive energy density. From Fig. 3, we can see that the EoS parameter remains staying negative throughout the graph. It lies in the region $(-1,0)$ representing the quintessence phase for the observational Hubble dataset. But in the case of BAO and Hubble+BAO datasets, it is showing the analogous results. It lies in the range $(-1,0)$ that is in the quintessence region in the early time, just after $z = 0$, it comes below -1 and stays in the phantom region that is below -1 in the late time. At the redshift $z = 0$, the value of the EoS parameter for this model are estimated to be $\omega_0 = -0.915$, $\omega_0 = -0.911$, and $\omega_0 = -0.912$ which are consistent with the previous studies [10–12].

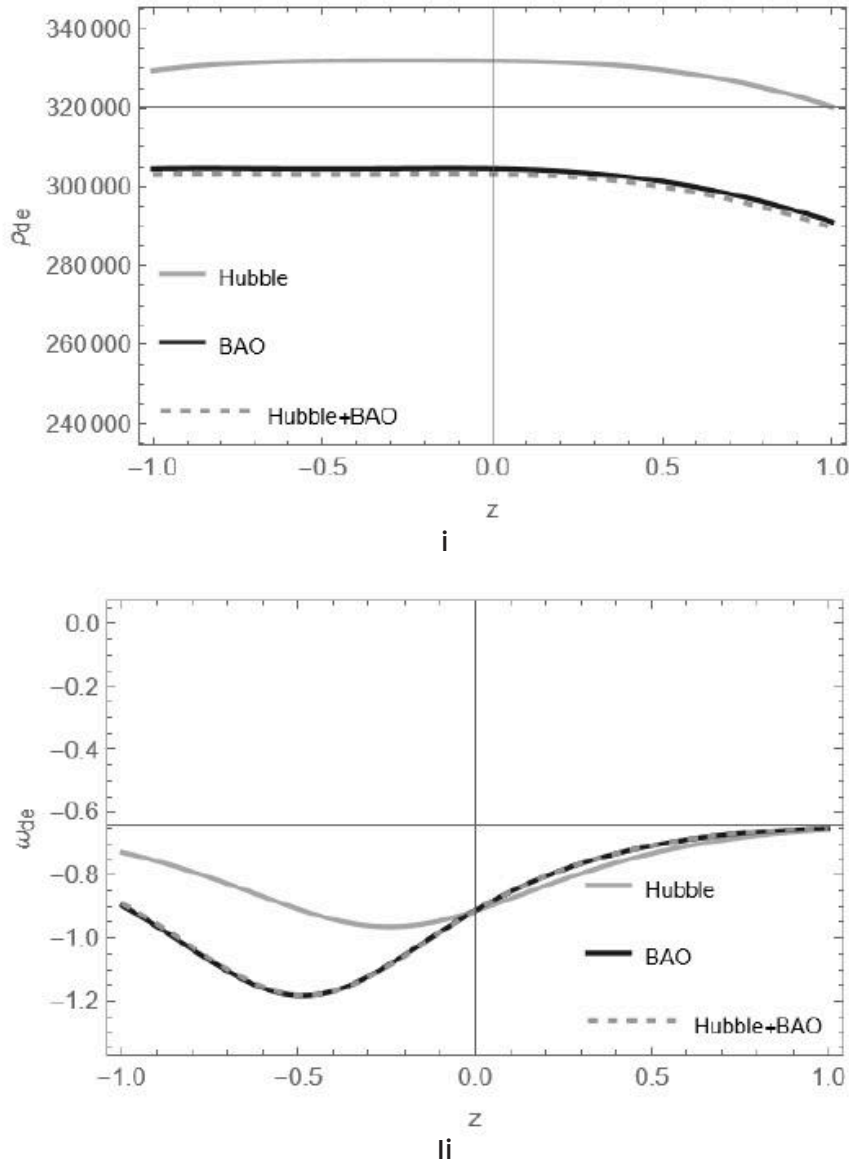


Fig. 3. (i) Left panel is the plot of energy density, and (ii) right panel is the plot of EoS parameter with parameter scheme $\alpha = 47.4$ and $n = 0.038$

Conclusion. In this study, we have reconstructed a cosmological model with accelerating behaviour emphasizing late-time evolution. In the context of TG, first the field equations got derived and to proceed for the dynamical study, a particular form of $f(T)$ inspired from a $f(R)$ theory is taken to determine the dynamical parameters. In order to describe the evolution, we considered a special Hubble parameter dependent upon three free parameters which got constrained through the MCMC approach using observational datasets such as Hubble, BAO and a combination of both. From the error bar plot of our Hubble parameter, we observed that the behaviour of the Hubble is consistent with the Λ CDM model. The graph of the Deceleration parameter is showing that the plot is staying negative throughout addressing the accelerating behaviour of our model. Where, the current values of both Hubble parameter and Deceleration parameter are carrying in the range range

[69.550, 72.76] and $[-0.53, -0.74]$, respectively with the values of the Deceleration parameter are in agreement with the previous literatures mentioned in the corresponding section. In order to get to know about the dynamical behaviour of our model, the EoS parameter got investigated which remained negative throughout the graph. For the Hubble dataset it is lying in the quintessence region, but for BAO and Hubble+BAO datasets it is lying in the quintessence region at the early time and just after $z = 0$, it is coming to the phantom while moving towards the late time. The estimated current values of the EoS parameter are consistent with the previous results which are covered in the respective section.

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