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Extended Dirac principle of superposition of evolution of time in non linear quantum processes

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Abstract. The Dirac's principle of superposition for evolution of time (DPSET) was developed to restore determinacy in the study of dynamic linear evolution of quantum system. Here we advance the formulation to non-linear dynamics by invoking the quantum noise-extended nonlinear Schrodinger equation into the four motions in time conjecture of the DPSET. We observe that in the resultant non-linear temporal evolution of the wavefunction, the linear DPSET can be recovered from the nonlinear domain in the limit of Rabi oscillations. We then delineate the potential linear and non-linear regimes for the time-reversal symmetry of the measurement problem. In conclusion, we postulate that the non-linear domain provides information for understanding abruptness and strangeness in quantum measurements.

Keywords: quantum measurement indeterminacy, Dirac's principle of superposition, evolution of time, non-linear dynamics, time reversal symmetry

Introduction. The search for quantum determinism is still an open challenge as we celebrate the first century of the formulation of quantum mechanics. This has made it difficult to reach a consensus interpretation of quantum mechanics [1–3]. There have been deluge of attempts to solve this measurement problem but yet the problem still lingers on [3–5]. One consensus is that the quantum state evolves deterministically under the Schrödinger Equation leading to superpositions: however, when it is measured, it is stochastically governed by the Born rule to collapse into a single outcome [3]. This dual evolutions of quantum states is the cause of the disharmony in the measurement problem.

In Ref [4], we formulated the Dirac's Principle of Superposition of evolution of time (DPSET), in an attempt to harmonize the outcomes of the dual evolutions. We demonstrated that the DPSET four possible time evolutions were likely to be in one-on-one correspondence with the motions in the linear quantum process. For brevity, the DPSET was used to successfully account for the various bonding of beryllium as a linear quantum process.

However, the DPSET evolution in time is unitary and therefore does not capture all the rich dynamics of indeterminacy. For example, there have been a num-

ber of studies on non linear quantum processes that need theoretical interpretation [5, 6]. Therefore, our plan in this paper is to advance the DPSET beyond the linear limit and then use it to investigate non linear quantum processes.

Theoretical framework. The theoretical motivation is that the DPSET can be illuminated by the susceptibility of a quantum system to fluctuations and noise which can be interpreted as abrupt changes in the behavior of the quantum process with time. We employ the formulation in Ref [5] on the susceptibility of a quantum system to fluctuations and noise, thus the quantum noise-extended NSE which models the behavior of quantum systems more accurately:

$$i \frac{\partial \psi}{\partial t} - \nabla^2 \psi + 2|\psi|^2 \psi - 2\rho^2 \psi + \sigma \psi \frac{d\mu}{dt} = 0 \quad (1)$$

$t \geq 0$, $r \in \mathbb{R}$, $\psi = \psi(r, t)$ is a complex variable, ρ and σ are parameters, $\frac{d\mu}{dt}$ represents the first derivative of the motional noise μ . Following the author [6], we have obtained variants wavefunction solutions for the stable non-linear Schrodinger equation:

$$|\psi_k(r, t_j^i)\rangle = \left(\frac{iB_0 \sqrt{\sigma^2 - 2\rho^2 - \frac{2V_n}{m}}}{\sqrt{2}B_0 \left(1 - \frac{2}{1 + A \exp(\pm i(r \pm r_n)) \sqrt{2(\sigma^2 - 2\rho^2 - \frac{2V_n}{m})}} \right)} \right) e^{\pm i[(r \pm r_n) + \sigma \mu] - i\Delta E t_j^i / \hbar} \quad (2)$$

where m is mass of particle, t_j^i is the evolution of time in the DPSET conjecture, V_n is the depth of optical lattice potential, r is particle position in 1-, 2- or 3-D, $r_n \leq 0$ is the shift in the particles position, $\hbar$ is the reduced Planck's constant, ΔE is transition energy between the two consecutive states, ψ_k is the wavefunction corresponding to the temporal evolution t_j^i .

Results and discussion. The extended results in Eq. (2) poses Stochastic time dependance, but the actual physical dynamics to be observed depends on the Energy, in agreement with the DPSET [4]. In the model (2), it can be deduced that the classical dynamics of the measurement dynamics can be recovered by squaring the wavefunction or by treating the convolution of the forward and reversed wavefunction in temporal evolution of times $t_j^i: |\psi_k(r, t_j^i)|^2$. Another quantum property of interest in both the linear and non-linear DPSET formalism is the time-reversal symmetry [6] denoted by the $e^{\pm i[(r \pm r_n) + \sigma \mu]}$ term in the wavefunctions which predict that the present state of a quantum system defined by both the usual forward-evolving wavefunction from the past and backward-evolving one from the future.

Conclusion. The non-linear DPSET formalism presented here shows promising quantum features for probing into problem of indeterminacy in quantum measurements due to abrupt and strange behavior inherent in quantum system dynamics.

The model presented here has been theoretically investigated and quantitatively shows intriguing features which include collapse, decoherence, decay, uncertainty, time-reversal symmetry and quantum transitions. We therefore propose this model as candidate for theoretical study of the recent negative time experiment and in the design and engineering of highly sensitive time measurement devices.

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