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Introduction to the Unimodular Kaluza — Klein theory

Kerner Richard^{1(*)}

richard.kerner@sorbonne-universite.fr

Julio C. Fabris²

julio.fabris@cosmo-ufes.org

¹LPTMC, Sorbonne University and CNRS, Paris, France

²UFES, Vitoria, Brazil

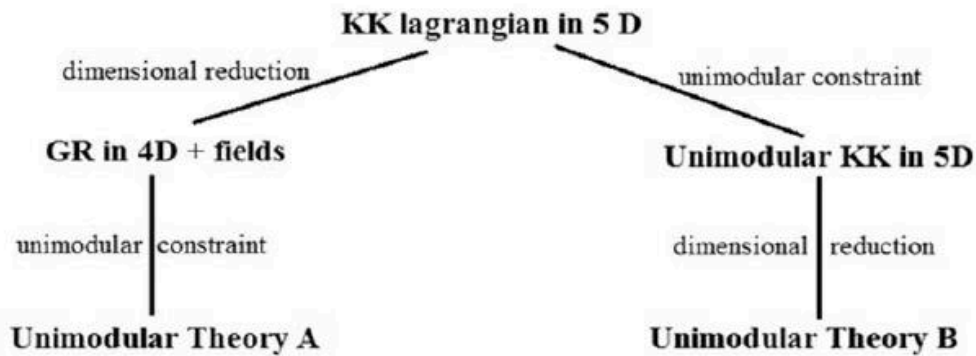
Abstract. Unimodular Gravity was first introduced by Einstein as early as in 1919. The extra unimodularity condition imposed on the pseudo-Riemannian metric seemed to be a promising alternative to the cosmological constant, by using the condition $\det(g) = 1$ as a Lagrange multiplier in the Einstein — Hilbert variational principle. In this talk, we shall investigate physical and mathematical aspects of extension of the unimodularity condition to the 5-dimensional Kaluza — Klein space. It turns out that when the unimodularity constraint applies to the five dimensional equations, it does not necessarily apply to the reduced equations in four dimensions. The reduction of the five dimensional equations to four dimensions leads to a quite unusual structure connecting gravity to a scalar field and ordinary matter, which stems from the five dimensional framework. Cosmological vacuum solutions of the effective four dimensional equations have been derived in our recent paper [1]. They describe a symmetrically bouncing universe. However, even if the four dimensional metric is free of singularities, the original five dimensional one becomes degenerate at the bounce since the modulus field associated with the fifth dimension vanishes at the transition point from the contracting to the expanding phase.

Keywords: Unimodular Gravity, Kaluza — Klein model, Cosmology, Bouncing solutions

Introduction. The unimodular gravity was viewed as another approach to the cosmological constant problem and to quantization of gravity. In fact, the unimodular condition implies that the resulting field equations, after solving the constraint on the determinant of the metric, are traceless. Due to this property, any cosmological constant term disappears from the field equations. However, it can reappear as an integration constant. One consequence of this property is that, for example, the possible connection of the cosmological constant, for example, with the vacuum energy is lost. Notice, moreover, that instead of imposing the determinant of the metric equal to 1, the unimodular condition can be generalized by imposing that the determinant of the metric is equal to the determinant of a given fiducial metric. In any case, the condition on the determinant implies that the theory is invariant by a restricted class of diffeomorphism, the transverse diffeomorphism. By generalizing the unimodular gravity to the Kaluza — Klein, we follow the lines sketched in our previous publication.

Our investigation of unimodular gravity in the context of the 5-dimensional Kaluza — Klein theory should be preceded by the following considerations. We are confronted with an alternative choice of the way we apply the unimodularity

condition. We can choose to impose it to the variational principle in 5 dimensions, find the solutions and then proceed to dimensional reduction, or perform dimensional reduction first, expressing everything in terms of 4-dimensional Riemannian metric and electromagnetic and scalar fields, and then proceed to variational principle in 4 dimensions with unimodular condition imposed on the 4-dimensional metric. The situation is illustrated by the following diagram:



In order to explore all the degrees of freedom present in the Kaluza — Klein setting, we choose to apply the unimodular condition directly to the 5-dimensional variational principle, leading to “theory B” on our diagram. Cosmological model based on this theory does not need cosmological constant, and can lead to bouncing Universe instead of singularity at the Big Bang.

Methods and materials; results. The methods are the usual ones in theoretical and mathematical physics: elaborating the model first, solving equations next, analyzing solutions’ properties and trying to find a physical interpretation.

Conclusion. We have explored cosmological solutions in the absence of electromagnetic field coming from the Kaluza — Klein structure, which is required to have isotropy in the three spatial dimensions of the external space. For the Kaluza — Klein unimodular gravity the reduction to four dimensions preserves the coupling to the electromagnetic field emerging from the five-dimensional metric. This is related to the traceless character of the electromagnetic field. The coupling of the scalar (modulus) field Φ associated to the fifth dimension, may possible bring new structures, e.g. static, spherically symmetric configurations, including Black Holes.

For the cosmological solution found here, we may also expect some peculiar features at perturbative level. Finally, the effective theory in four dimensions, and its coupling to matter emerging from unimodular Kaluza-Klein theory, must be developed further since they correspond to a new implementation of the coupling of the scalar field and gravity after reduction to the usual four-dimensional space-time.

References

- [1] Fabris J.C., Kerner R. A unimodular Kaluza — Klein theory. *Physica Scripta*, 2024, vol. 99 (8), art. no. 085042.