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A New Type I Cosmological Model with Singularity admits the Canonical Killing Tensor Forms

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Abstract. Investigating spacetimes with hidden symmetries is a promising approach to study “realistic” mathematical models enhances our understanding of gravitational phenomena. In this work initiated by the assumption of existence of the canonical Killing tensor forms we explore a new directive regarding the null tetrad transformations in extracting (algebraically) general Einstein solutions. Capitalizing on the (anti-)symmetric null tetrads concept a new Petrov type I solution emerges describing a cosmological model with a curvature singularity in vacuum with cosmological constant Λ .

Keywords: Petrov Type I, Type D, Canonical Killing tensor forms, Cosmological model, Cosmological Singularity

Introduction. The acquiring of the exact solutions of Einstein’s equations relies strongly on mathematical assumptions such as symmetries, potentially leading to (algebraically)¹ general solutions in the most favorable cases. The Einstein's Field Equations (EFE) is not a solvable system of equations without mathematical assumptions regarding symmetries. Hence, our investigation focuses on the spacetimes with hidden symmetries enabled by the Canonical Killing Tensor Forms (CKTF) for two reasons 1) not only because we scope to investigate the underlying role of hidden symmetry of geodesics of the most general type 2) but also to transform the over-determined system of equations to a solvable one adding the Integrability Conditions (IC) of Killing tensor.

The work of Hauser-Malhiot [1] in electrovacuum with Λ served as a paradigm for us. Notably, the CKTF are generalizations of the Hauser-Malhiot's form. In our previous work we generalized the study of spacetimes admitting an irreducible Killing tensor by acquiring the CKTF [2] in line with the work of Churchill [3]. Among the solutions obtained, initiating by $K_{2\mu\nu}$ and applying a general null tetrad transformation we were able to find and characterize algebraically a new type D solution characterized by a non-geodesic, shear-free and diverging null congruence.

¹ The use of this parenthesis indicates our intention to investigate the characteristics of a solution not only in terms of the possible reductions that may lead to special cases but also in terms of its algebraic characterization (Petrov types).

Along these lines, the resolution of Newman-Penrose Field equations and IC of $K^2_{\mu\nu}$ assuming the transformation (4) yields a new Petrov type I with $\Psi_0\Psi_4 \neq 9\Psi_2^2$ describing a cosmological model with a non-geodesic, shear-free and diverging null congruence with its hidden symmetry in non-null geodesics enabled by $K^2_{\mu\nu}$ Canonical Killing tensor form. It should be noted that the Weyl component of the solution, where $K = K_1 + iK_2$; $\Pi = \Pi_1 + i\Pi_2$ are constants of integration, has the form

$$2\Psi_2 = 2\Lambda - \frac{6\Lambda}{\tanh^2\left(\frac{-1}{2}\sqrt{\frac{6\Lambda}{\Pi\bar{\Pi}}}\left[\left(\Pi_1 + \frac{V_1\Pi_2}{V_2}\right)x + \frac{\Pi_2}{V_2}\right]\right)} - \frac{K\bar{K}\cosh^2\left(\frac{-1}{2}\sqrt{\frac{6\Lambda}{\Pi\bar{\Pi}}}\left[\left(\Pi_1 + \frac{V_1\Pi_2}{V_2}\right)x + \frac{\Pi_2}{V_2}\right]\right)}{\left[\left(K_1 + \frac{V_1K_2}{V_2}\right)x + \frac{K_2}{V_2}\right]^2};$$

where $x, y \rightarrow 0$ then $\Psi_2 \rightarrow \infty$ presenting a curvature singularity.

Conclusion. This work presents the outcomes of our investigation of spacetimes admitting the Canonical forms of Killing tensor. Applying two different kinds of null tetrad transformations we were able to derive two algebraically different solutions (Types D, I) and propose new directive. This directive suggests that there is a preferability in null tetrads transformations based on the (anti-)symmetric null tetrads concept correlating the spin coefficients between themselves instead of boosts and spatial rotations. By using this directive we were able to extract a new Petrov type I solution with $\Psi_0\Psi_4 \neq 9\Psi_2^2$ where presenting a curvature singularity.

References

- [1] Hauser I., Malhiot R.J. Forms of all spacetime metrics which admit [(11) (1,1)] Killing tensor with nonconstant eigenvalues. J. Math. Phys., 1978, vol. 19 (1), pp. 187–194. <https://doi.org/10.1063/1.523523>
- [2] Kokkinos D., Papakostas T. The Study of the Canonical forms of Killing tensor in Vacuum with Λ . Gen Relativ Gravit, 2024, vol. 56, art. no. 134. <https://doi.org/10.1007/s10714-024-03321-w>
- [3] Churchill R.V. Canonical forms for symmetric linear vector functions in pseudo Euclidean space. Trans. Amer. Math. Soc., 1932, vol. 34, pp. 784–794. <https://doi.org/10.2307/1989429>
- [4] Churchill R.V. Canonical forms for symmetric linear vector functions in pseudo Euclidean space. Trans. Amer. Math. Soc., 1932, vol. 34, pp. 784–794. <https://doi.org/10.2307/1989429>
- [5] Debever R. On typed expanding solutions of Einstein — Maxwell equations. Bulletin de la Societe mathématique de Belgique, 1971, vol. 23, pp. 360–376.