

UDC 524.882

## Constraining Gauss — Bonnet Parameter and Radii of Compact Stars in the Mass-gap Using Realistic Equation of States in 4D-EGB Gravity

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**Abstract.** In this paper, we have investigated the effects of Gauss-Bonnet (GB) extension on Einstein's gravity in 4-dimensions. Four different EoS such as SLy4, APR4, FPS and Togashi are considered. These EoS are used to model the massive neutron stars like PSR J2215+5135, PSR J1959+2048, PSR J1810+1744, PSR J0740+6620 and GW190814, which are in the neutron star-black hole (NS-BH) mass gap. We have calculated the mass and radius through the M–R curves for different values of the GB-coupling parameter for four different EoSs to examine the upper limit of masses that fall within the mass-gap. Our result shows that the positive branch of  $\alpha$  is suitable for explaining the existence of massive neutron stars in the mass-gap. Bondi's and static stability criteria have confirmed the stability of these massive neutron stars. In fact, an increase in GB-coupling strength improves the stability of these massive neutron stars. Furthermore, the surface redshift of these massive neutron stars is below the maximum limit, i. e.  $Z_s < 2$ . Finally, we have used the M–R curves with the observed masses to predict the radii of PSR J2215 + 5135, PSR J1959 + 2048, PSR J1810 + 1754, PSR J0740 + 6620 and GW190814 in GR and 4D-EGB gravity. The predicted radii are in agreement with the constraints from the GW170817 event. Lastly, the maximum observed NS mass is fitted in the M–R curves implying that the range of  $\alpha$  should be somewhere around 0 to 5 [km<sup>2</sup>].

**Keywords:** stars, neutron-stars, massive-black hole physics, analytical- equation of states

**Introduction.** Post Einstein general relativity (GR), there were lot of interest to explore the field of relativistic astrophysics. In 1931, Landau theorized the possibilities of compact objects denser than white dwarfs with supra nuclear densities. Tolman [1] and Oppenheimer & Volkoff [2] developed the equation of hydrostatic equilibrium in the context of GR known as the TOV equations, which has been used to determine the maximum mass of compact objects. Unfortunately, determining the maximum masses of compact objects like neutron stars (NSs) are still

a mystery in modern astrophysics. In addition, the NS-BH mass gap ( $\sim 2\text{--}5M_{\odot}$ ) which is between the maximum mass of neutron stars and the least massive black holes (BH) is still unknown [3]. It has been argued that there are no compact objects between the mass gap. But recent observations have shown that the mass-gap as for compact objects as  $\sim 2.50\text{--}2.67M_{\odot}$  [4]. We are in tending to investigate the mass-gap which can be done in two ways, such as extending GR or exploring more stiff equation of state (EoS) viz. Sly4, APR4, FPS and Togashi EoS.

GR has effectively addressed many aspects of the universe, but it has limitations, such as its inability to explain the NS-BH mass gap and its restriction to four dimensions. To address these shortcomings, we can modify the gravitational component of the Einstein-Hilbert action, leading to the development of modified gravity theories. We consider the gravitational part of the Einstein-Hilbert action with the generalization of Einstein's gravity to higher  $D$  dimensions, like Lovelock gravity [5, 6], which also can revert back to Einstein's theory in  $D = 4$ . In higher dimensions the Einstein-Gauss-Bonnet (EGB) gravity [7] can be formed by including the Gauss-Bonnet term in the Einstein-Hilbert term. Though the GB term is topological in 4D, but can be redefined by a regularization technique by introducing quantum corrections to Einstein's gravity. In this theory, Lovelock's theorem was bypassed, and as a result, the contribution of the GB term in gravitational dynamics becomes non-negligible, which enables 4D-EGB gravity.

**Methods and materials; results.** It is necessary to solve the TOV equations to explore the connections between different physical quantities, such as the  $M\text{--}R$  curve. To numerically solve these equations, we apply the following initial conditions at  $r = 0$  in the TOV equations as  $p(0) = p_c$  and  $m(0) = 0$ . As the pressure-density relationship originates from the EoS, we can immediately move onto solving the TOV equations. For numerical integration, we utilized the Runge-Kutta 45 (RK45) method available in the 'scipy.integrate' package in Python. The neutron star's surface can be determined by examining the pressure, and the hydrostatics equation indicates that the pressure decreases as the radius,  $r$  increases. We define the star's surface at the point where the pressure,  $p(r)$  approaches zero, specifically at  $p(r_s) \approx 0$ , where  $r_s$  denotes the radius of the star. For the mass of the star, we calculate by using mass gradient equation. This equation accumulates the mass of the stars until it reaches  $r_s$ .

**Conclusion.** In this paper, we analyzed 4D-EGB gravity with realistic four EoSs namely SLy4, APR4, FPS and Togashi. By solving the modified TOV-equation, we have generated the  $M\text{--}R$  curve for the SLy4 EoS while varying the GB-coupling parameter  $\alpha$ . Further, we also compared the  $M\text{--}R$  curves for GR ( $\alpha = 0$ ) and 4D-EGB ( $\alpha = 5$ ) for the 4-EoSs. From this, we have found that the positive branch of  $\alpha$  is useful in constraining the radii of the recently observed massive neutron stars such as PSR J2215+5135, PSR J1959+2048, PSR J1810+1744, PSR J0740+6620, and GW19081, ensuring that the curve remains outside the unstable region. This analyses, implies that the 4D-EGB gravity allows more massive neutron stars to exist compared to the GR counterpart with the same matter composi-

tion/EoS. The stability of such massive neutron stars has been proven by the Bondi's and static stability criteria while the modified TOV equation itself ensures the equilibrium condition. Further, it is also pointed out that the stability of compact star improves with increase in  $\alpha$  values, implying that 4D-EGB gravity allows to exist more massive compact stars in the mass-gap. We also plotted  $Z_s - M$  for the four EoSs and satisfy the  $Z_{s\max} \lesssim 2$  limit. It can also be noted that all four EoSs are stiff enough to hold massive and stable configurations when the central density is above  $10^{14}$  g/cm<sup>3</sup> where the adiabatic index is above  $4/3$ .

From the above analyses, we have used these realistic EoSs to constrain the radius and the range of Gauss Bonnet coupling strength from the upper mass limit which are below the unstable region. Constraining of the radii can be done from the error bars of the observed masses. Here, the radii are larger in 4D-EGB than in the GR counterpart. The  $M$ -max also increases for more GB-coupling strength in all EoSs. In all EoSs, the radius range of compact stars ranges from 10.63–12.09 km. One of the most compact objects observed so far has been predicted in GW190814 event, which is expected to be a merger of a very massive neutron star of mass  $2.5\text{--}2.67M_\odot$  and a light black hole of mass  $22.2\text{--}24.3M_\odot$ . This massive neutron star in the NS-BH mass gap cannot be accommodated with respect to SLy4, FPS, APR4 and Togashi in the GR limit, however, in 4D-EGB it can exist with a radius of 10.63 km (SLy4), 11.46 km (APR4) and 11.66 km (Togashi) with  $\alpha = 5$ . Furthermore, from the more recent gravitational waves observation of NS-NS merger in GW170817 event, a stringent constraint on radius of has been found where a NS of mass  $1.6M_\odot$  and  $1.4M_\odot$  should have a radius larger than  $10.68^{+0.15}_{-0.04}$  km and  $11.0^{+0.9}_{-0.6}$  km respectively. These stringent constraints are clearly satisfied in the  $M$ - $R$  curves where a  $1.6M_\odot$  NS have at least 10.5 km and for  $1.4M_\odot$  at least 11 km radius. Finally, we can use these  $M - R$  curves to constrain the coupling strength  $\alpha$  by observing the minimum and maximum values of  $\alpha$  need to fit the observed masses. It has been found that the massive NS in GW190814 i.e.  $2.5\text{--}2.67M_\odot$  can be fitted with  $\alpha = 5$ . Hence, the range of  $\alpha$  should be approximately between 0 and  $5\text{ km}^2$  (according to the maximum NS mass observed yet).

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