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## Massive scalar and Dirac Perturbations of a four-dimensional black hole in the presence of perfect fluid dark matter

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**Abstract.** In this paper, we study quasinormal modes and absorption cross sections for the (3+1)- dimensional Bardeen black hole surrounded by perfect fluid dark matter. Hence, this paper will focus on the massive scalar field perturbations and massless Dirac field perturbations. To compute the quasinormal modes by using the semi-analytical WKB method and the Pöschl — Teller method. We show our results, in figures, and we confirm that all the modes are found to be stable against both (massive) scalar and Dirac perturbations. The absorption cross section is also computed.

**Keywords:** black holes, perfect fluid dark matter, QNMs, cross section, WKB method, Pöschl — Teller method, perturbations

**Introduction.** Quasinormal modes (QNMs) of a black hole are the characteristic oscillations that arise when the spacetime is perturbed. These oscillations are described by solutions to wave equations in the black hole background, with boundary conditions enforcing purely ingoing waves at the horizon and outgoing waves at infinity. Their frequencies are complex, where the real part represents the oscillation frequency and the imaginary part determines the damping rate due to gravitational wave emission. QNMs form the resonant “fingerprint” of a black hole and play a key role in the dynamics of mergers and relaxation processes.

Computing quasinormal modes is crucial for several reasons. In gravitational wave astrophysics, QNMs dominate the ringdown phase following black hole mergers, allowing us to infer their parameters and test general relativity. Additionally, in quantum gravity and the context of the AdS/CFT correspondence, QNMs are linked to the relaxation time of thermal quantum states. Methods such as continued fraction expansions or semiclassical approximations provide highly accurate QNM spectra, offering powerful tools to probe modified gravity theories.

The cross section is a fundamental quantity in scattering theory and describes the effective probability that an incoming wave interacts with an object, such as a black hole. In the context of wave scattering, the cross section determines how much energy is absorbed, reflected, or refracted by the black hole. In particular, the absorption cross section and the effective scattering cross section are directly related to phenomena such as gravitational lensing and superradiance, playing a key role in the indirect detection of black holes and the testing of quantum gravitational effects.

In this work we have computed the QNMs of black holes in the presence of perfect fluid dark matter using two complementary methods, the first being the well-known WKB approximations and the second being the Pöschl — Teller fitting method. As for the absorption cross section, we used the 3rd order WKB approach to compute reflection, transmission coefficients and partial absorption cross sections.

**Methods and materials; results.** The WKB method is a semiclassical technique widely used to calculate quasinormal modes of black holes when the effective potential has a well-defined barrier with a single maximum. In this approach, the wave equation is treated as a Schrödinger-like equation, and its solutions are approximated in terms of oscillatory and exponential exponentials in classically allowed and forbidden regions, respectively. To determine the QNM frequencies, a connection condition is imposed between these regions, leading to a quantization condition based on derivatives of the potential at its peak. Improved methods, such as the Schutz-Will approach or higher-order WKB expansions, have provided increasingly precise QNM frequencies for different types of black hole perturbations. Depending on the order, the WKB method provides a good approximation to the corresponding QNMs of black holes. We then used this method to make progress, investigating different approximation orders.

Alternatively, the Pöschl — Teller potential is an exactly solvable model used to obtain approximate analytical solutions for black hole quasinormal modes. This potential takes the form

$$V(x) = V_0 \operatorname{sech}^2(\alpha x),$$

where  $V_0$  and  $\alpha$  are adjustable parameters that mimic the behavior of the actual effective potential near its maximum. The advantage of this model is that its solutions are known, leading to an analytical expression for QNM frequencies, providing insight into their dependence on system parameters. While it does not exactly describe the spectrum of a real black hole, it serves as a useful estimate and a consistency check for numerical methods and semiclassical approximations like WKB.

Finally, the absorption cross section is calculated using the 3rd order WKB approach. An analysis of the reflection and transmission coefficients as well as the partial absorption cross sections is performed.

**Conclusion.** Our results show that for this black hole solution, both the massive scalar perturbation and the Dirac perturbations maintain the stability of the black hole. In other words, our figures and tables confirm the stability, and then we conclude that this background is stable against both types of perturbation.

## References

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