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Exploring entropy in de sitter space using the covariant Euclidean approach

Singh Raghvendra (*)

raghavendra@ariel.ac.il

Ariel University, Ariel, Israel

Abstract. We derive the Euclidean entropy of four-dimensional de Sitter spacetime by integrating the Kodama-observer anomalous entropy density over the compact Euclidean section, obtained through a covariant Wick rotation discussed in [2]. Starting from the comoving metric $ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2 d\Omega_2^2)$ and enforcing the Kodama trajectory $re^{Ht} = C$ [2], we evaluate the anomalous entropy density $S_{\text{anomalous}} = (3H^2)/(8\pi G)$ and show that the corresponding four-volume element reduces, after an exact coordinate transformation, to that of a round four-sphere of radius H^{-1} . The resulting total entropy $S_{\text{anomalous}} = \pi/(GH^2)$ reproduces precisely the Bekenstein-Hawking entropy $(A/4G)$ associated with the cosmological horizon. This calculation demonstrates that the anomalous entropy approach [3], grounded in the Euclidean Quantum Gravity framework, yields correct and physically consistent results.

Keywords: black hole entropy, wick rotation, dynamical spacetime, kodama observer

Introduction. Euclidean Quantum Gravity offers a non-perturbative route to gravitational thermodynamics by relating the semiclassical partition function to Euclidean manifolds [4]. Conventional methods rely on static spacetimes with analytic continuations, however, these methods are inadequate for dynamical, non-stationary geometries. The covariant Wick-rotation approach developed by R. Singh, D. Kothawala [1] addresses this gap by defining Euclidean metrics through a positive-definite combination $\widehat{g}^{ab} = g^{ab} - \Theta u^a u^b$, where u^a is a nowhere vanishing timelike vector field and Θ is the interpolating function between two regimes generate Euclidean metric for $\Theta = -2$ in covariant way. We apply this technique specifically to the Kodama observer in de Sitter spacetime, enabling a robust calculation of horizon entropy.

Methods and materials; results. We apply the covariant Wick-rotation formalism to the de Sitter spacetime metric, employing Kodama observers characterized by trajectories defined by $re^{Ht} = \text{constant}$. The resulting Euclideanized metric contains cross-terms which are eliminated via the transformation $w = Hre^{Ht}$, revealing explicitly the geometry of a four-sphere. The anomalous entropy density, defined by $(16\pi G)S_{\text{anomalous}} = -2G_{ab}u^a u^b + \widehat{R}$, evaluates to $S_{\text{anomalous}} = 3H^2/(8\pi G)$. Integration over the Euclidean manifold yields the total entropy $\pi/(GH^2)$, exactly matching the standard Bekenstein — Hawking formula [5].

Conclusion. In conclusion, anchoring the covariant Wick rotation to a general, nowhere-vanishing timelike observer field — exemplified here by the Kodama

congruence — offers a powerful, self-consistent framework for computing horizon entropy in dynamical spacetimes, as explicitly demonstrated for de Sitter space. This approach underscores the universality and robustness of gravitational entropy calculations within the Euclidean formalism, extending its applicability well beyond stationary solutions.

References

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