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Absolute Unified Mathematics of Quantum Gravitation

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Abstract. The paper comprises a generalized solution to non-linear PDEs, mainly interpreted in the gravitation field equation [1]. The author shows that gamma matrices (Spinors) are embedded within Christoffel symbols. This approach is usable for calculating geodesic as: $-\Gamma_{00}^{\alpha}c^2 = (2\pi m_0 c^3 I/\sqrt{-1}h)$, for a weak quantum gravitation field. Hence, it concludes the followings: (a) 24 mathematical terms of second-order, as an integral part of Ricci tensor; and (b) unification of all space-time geometries [3, 4].

Keywords: quantum gravity, general relativity, clifford algebra, ricci tensor, ricci flow, geodesic

Introduction. Absolute Unified Mathematics of Quantum Gravitation, “AUM”, is developed to offer a complete solution to the following existing problems. These includes: (I) quantum gravity and general relativity; (II) singularity and dark energy; and (III) many more. Its key objective is to incorporate Spin (gamma matrices) within Christoffel Symbols, as discussed in the next section. This is of uttermost relevance for analyzing the Spin-Dynamics of the gravitational field. Scientifically, it discloses 24 second-order partial derivative terms (Quantum Gravitational Waves), as an integral part of Ricci tensor.

Methods and materials; results. To derive the embedded quantum fields within Christoffel, “ $\Gamma_{ij,m}$ ”, I started with a covariant differentiation, “ ∇_j ”, of a fundamental tensor, “ g_{im} ”, as shown in equation (2) [1]. These equations are acquainted with first order covariant derivatives of gamma matrices, “ γ_i ” and “ γ_m ”. According to the Clifford Algebra, these matrices are also referred as the basis vectors of fundamental tensor, “ g_{im} ”. These basis vectors, “ γ_i ” and “ γ_m ”, are fundamentally known by equation; $g_{im} = \frac{\gamma_i \gamma_m + \gamma_m \gamma_i}{2}$ [2].

$$\frac{\partial g_{im}}{\partial x^j} = \frac{1}{2} [\{\nabla_j \gamma_i\} \gamma_m + \gamma_i \{\nabla_j \gamma_m\} + \{\nabla_j \gamma_m\} \gamma_i + \gamma_m \{\nabla_j \gamma_i\}] + g_{pm} \Gamma_{ij}^p + g_{ip} \Gamma_{mj}^p \quad (1)$$

The first order covariant derivatives, “ $\nabla_j, \nabla_i, \nabla_m$ ”, of these fundamental tensors, “ g_{im}, g_{jm}, g_{ij} ”, are derived like equation (1). Further, I derived a torsion-free Christoffel symbols of second kind, “ Γ_{ij}^{α} ”. This was pursued by substituting the partial derivatives of fundamental tensors, “ $\frac{\partial g_{im}}{\partial x^j}, \frac{\partial g_{jm}}{\partial x^i}, \frac{\partial g_{ij}}{\partial x^m}$ ”, into the well-known equation of “ $\Gamma_{ij}^{\alpha} = \frac{1}{2} g^{\alpha m} \left[\frac{\partial g_{im}}{\partial x^j} + \frac{\partial g_{jm}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^m} \right]$ ” [1]. With further simplification, I have discovered equation (2) [3, 4].

$$\Gamma_{ij}^{\alpha} = g^{\alpha m} \frac{1}{4} \left(\gamma_i \left(\frac{\partial \gamma_m}{\partial x^j} - \frac{\partial \gamma_j}{\partial x^m} \right) + \left(\frac{\partial \gamma_m}{\partial x^j} - \frac{\partial \gamma_j}{\partial x^m} \right) \gamma_i + \gamma_j \left(\frac{\partial \gamma_m}{\partial x^i} - \frac{\partial \gamma_i}{\partial x^m} \right) + \right. \\ \left. + \left(\frac{\partial \gamma_m}{\partial x^i} - \frac{\partial \gamma_i}{\partial x^m} \right) \gamma_j + \gamma_m \left(\frac{\partial \gamma_i}{\partial x^j} + \frac{\partial \gamma_j}{\partial x^i} \right) + \left(\frac{\partial \gamma_i}{\partial x^j} + \frac{\partial \gamma_j}{\partial x^i} \right) \gamma_m \right); \quad (2)$$

$$\frac{d^2 x^{\alpha}}{(dx^0)^2} = -[\Gamma_{00}^{\alpha}] c^2 - \\ - \frac{1}{2} \left[\left(\gamma_0 - (\gamma_1 + \gamma_2 + \gamma_3) \right) \frac{\partial \gamma_0}{\partial x^0} + \frac{\partial \gamma_0}{\partial x^0} (\gamma_0 - (\gamma_1 + \gamma_2 + \gamma_3)) \right. \\ \left. + \left(\gamma_0 \left(\frac{\partial}{\partial x^0} (\gamma_1 + \gamma_2 + \gamma_3) - (\nabla) \gamma_0 \right) + \left(\frac{\partial}{\partial x^0} (\gamma_1 + \gamma_2 + \gamma_3) - (\nabla) \gamma_0 \right) \gamma_0 \right) \right] c^2 = \\ = - \left[\frac{2\pi}{\sqrt{-1}h} \left(\frac{\hat{E}}{c} - \hat{P}_x - \hat{P}_y - \hat{P}_z \right) I \right] c^2; \quad (3)$$

$$R_{ij} = - \left[\frac{\partial g^{\alpha m}}{\partial x^{\alpha}} \Gamma_{ij,m} + g^{\alpha m} \frac{\partial \Gamma_{ij,m}}{\partial x^{\alpha}} \right] + \left[\frac{\partial g^{\alpha m}}{\partial x^j} \Gamma_{i\alpha,m} + g^{\alpha m} \frac{\partial \Gamma_{i\alpha,m}}{\partial x^j} \right] + \Gamma_{\beta j}^{\alpha} \Gamma_{i\alpha}^{\beta} - \Gamma_{\beta\alpha}^{\alpha} \Gamma_{ij}^{\beta} \quad (4)$$

Using equations (3), I calculated geodesic for a weak quantum gravity, as shown by equation (3). As a result, it illustrates “energy-momentum relation” within geodesic. Hence, validating equation (2). Additionally, the Christoffel symbols: $\Gamma_{ij,m}$, $\Gamma_{i\alpha,m}$, $\frac{\partial \Gamma_{ij,m}}{\partial x^{\alpha}}$, $\frac{\partial \Gamma_{i\alpha,m}}{\partial x^j}$, $\Gamma_{\beta j}^{\alpha} \Gamma_{i\alpha}^{\beta}$, and $\Gamma_{\beta\alpha}^{\alpha} \Gamma_{ij}^{\beta}$; given in equation (4) are also derived using equation (2), respectively [3, 4].

Conclusion. In this framework, I have concluded the root characteristics of Quantum Gravitation. These are elucidated through quantum fields of skew-symmetric, “ $\left(\frac{\partial \gamma_m}{\partial x^j} - \frac{\partial \gamma_j}{\partial x^m} \right)$ ” and “ $\left(\frac{\partial \gamma_m}{\partial x^i} - \frac{\partial \gamma_i}{\partial x^m} \right)$ ”, and symmetric, “ $\left(\frac{\partial \gamma_i}{\partial x^j} + \frac{\partial \gamma_j}{\partial x^i} \right)$ ”, nature. These fields are found embedded within Ricci curvature tensor, “ R_{ij} ” based on equations (2) and (4) [3, 4].

References

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