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The cosmic microwave background radiation and the cosmic neutrino background. Vacuum Structure and Quantum Field Theory

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Abstract. We report an extended summary of the results presented in the (open access) article in Ref. [1]. The thermal states at the temperature of the cosmic microwave background (CMB) are analyzed and it is shown that the vacuum expectation value of the energy momentum tensor density of photon fields reproduces the energy density and pressure of the CMB. The computations are done in the framework of the Thermo Field Dynamics (TFD). Neutrinos and thermal states at the temperature of the neutrino cosmic background (CNB) are also considered. Consistency with the estimated lower bound of the sum of the active neutrino masses is verified. The fractal self-similar structure of the thermal radiation is also discussed and related to the coherent structure of the thermal vacuum.

Keywords: cosmic microwave background, cosmic neutrino background, thermal vacuum condensate, thermo field dynamics, non-unitary time evolution, fractal self-similar structure

Introduction. By following Ref. [1], we discuss the thermal vacuum condensate for scalar and fermion fields with reference to the temperatures of 2.72548 ± 0.00057 K [2] characteristic of the cosmic microwave background (CMB) filling almost uniformly the observable universe and appearing as a radiation left over from an early stage in its expansion [3-6]. The anisotropies contained in the spatial variation in the spectral density are attributed to small thermal variations, presumably generated by quantum fluctuations of matter [1, 4, 5].

The expectation value on the thermal vacuum of the energy momentum tensor density of photon fields is computed and the energy density and pressure of the CMB is obtained [1].

Together with the CMB, there is indirect evidence of the existence of the cosmic neutrino background (CNB) which represents the universe's background particle radiation composed of neutrinos (relic neutrinos) [7,8], whose estimated temperature is roughly 1.95 K [6]. The study of the thermal vacuum condensate is extended also to the CNB case by computing the energy density of the neutrino thermal vacuum. The fractal self-similar structure of the thermal vacuum is finally discussed [9, 10]. The computations are carried on in the formal framework of the Thermo Field Dynamics [11, 12].

Methods and materials; results. A distinctive difference between quantum field theory (QFT) and quantum mechanics (QM) is that in QFT there are infinitely many “spaces of the system states” (unitarily inequivalent representations of the canonical (anti-)commutation rules (CCR or CAR)). The observables have differ-

ent values in each of the representations; there are many “vacuum states” which differ among themselves by a different “condensate” density: each one of them thus describes a different dynamical regime for the system. QFT is therefore adequate, contrarily to QM (the von Neumann theorem in QM), to describe different phases in which the system may live depending on physical boundary conditions and parameters, such as, for example, the temperature.

In view of such remarks, the mathematical framework which is adequate to the problem we discuss is therefore QFT. We will use $\hbar = 1 = c$.

In QFT at finite temperature, in the formalism of thermo-field dynamics (TFD) [11,12], the “thermal vacuum state” $|0(\theta)\rangle$ adopted for bosonic CMB (photon) modes is a two-mode time dependent generalized SU (1, 1) coherent state, condensate of pairs of a_k and b_k quanta. Here $\theta = \theta(\beta)$, with $\beta = 1/(k_B T)$ and T is the temperature, denotes the “Bogoliubov angle”, or phase, controlling the density of the condensate [1].

The operator b_k is an auxiliary boson mode, commuting with a_k , necessary in order to compute thermal averages of the observable quantities. One then finds that the minimization of the free energy leads to the Bose-Einstein distribution function for a_k . The expectation value of the operator number of a_k computed in $|0(\theta)\rangle$ is indeed $N(a_k) = 1 / (\exp(\beta \omega_k) - 1)$.

At $T = T_\gamma$, we obtain the observed black body spectrum of the CMB. The conclusion is that the cosmic microwave “background” is described by the coherent condensate vacuum $|0(\theta)\rangle$ with $\theta(\beta_\gamma)$, for $\beta_\gamma = 1/(k_B T_\gamma)$.

At thermal non-equilibrium, with temperature changing in time, $T = T(t)$, the system evolves through unitarily inequivalent spaces (representations), since at different time $t \neq t'$, for $T(t) \neq T(t')$ (i. e. $\beta \neq \beta'$), it is $\langle 0(\theta') | 0(\theta) \rangle \rightarrow 0$ in the infinite volume limit $V \rightarrow \infty$. One finds that such a time evolution is controlled by the entropy operator (the arrow of time). This describes the (nonunitary) Universe evolution through its ages.

The off-diagonal terms of the energy momentum tensor $T^{\mu\nu}$, for Maxwell, scalar and Dirac fields, are found to be zero, i. e. $T^{ij}(x) = 0$, for $i \neq j$. Therefore, the vacuum condensate is homogenous and isotropic and behaves as a perfect fluid.

The energy density ρ and pressure p of the vacuum condensates, at a given time, are given by the (0, 0) and (j, j) components of the energy-momentum tensors on the vacuum $|0(\theta, z)\rangle$, with z denoting the red shift of the universe (at the present it is $z = 0$). In the photon fields case, computations lead [1] to the state equation of the radiation $w(z) = p(z)/\rho(z) = 1/3$.

At the present CMB temperature, the thermal vacuum energy density is obtained: $\rho_\gamma = 2 \times 10^{-51} \text{ GeV}^4$, coinciding with the energy density of the CMB [6].

Negligible values of ρ are obtained for boson masses $m \gg 10^{-3} \text{ eV}$. Therefore, apart from the contribution given by hypothetical particles (axion-like particles), the thermal vacuum contribution of bosons to the universe energy is completely negligible with respect to ρ_γ .

In the fermion case, the thermal vacuum contribution to the vacuum energy density $\rho_F(z)$ and to the pressure $p_F(z)$, for masses $m \leq k_B T_\gamma$, has the maximum value of ρ_F at $T = T_\gamma$: it is $\rho_F \sim 1.6 \times 10^{-51} \text{ GeV}^4$. The state equation is $w_F \sim 1/3$.

The vacuum condensate of more heavy fermions gives negligible contributions to the universe energy. Therefore, only the thermal vacuum condensate of sub-eV massive particles, like the neutrinos, is relevant.

We consider then the expressions of $\rho_F(z)$ and $p_F(z)$, with $T = T_\nu = 1.95 \text{ K}$ and the sum on the three neutrino fields with masses m_i . For masses $m_i \sim 10^{-4} \text{ eV}$, the maximum value of the energy density is found [1] $\rho_\nu \sim 10^{-51} \text{ GeV}^4$, with state equation $w_\nu \sim 1/3$.

If one assumes that $\rho_\nu \leq \rho_\gamma$, and $m_{\nu,1} \sim 10^{-3} \text{ GeV}$ represents the mass of the lighter neutrino, and $\Delta m_{12}^2 = 8 \times 10^{-5} \text{ GeV}^2$, $\Delta m_{23}^2 = 2.7 \times 10^{-3} \text{ GeV}^2$ (the hierarchical neutrino model), we derive $\Sigma m_\nu = 6 \times 10^{-2} \text{ GeV}$, which is in agreement with the estimated lower bound on the sum of the three neutrino masses. The neutrinos with masses $m_{\nu,2}$ and $m_{\nu,3}$ give negligible contributions to the energy density ρ_ν .

Finally, the thermal vacuum $|0(\theta)\rangle$ has fractal self-similar structure. This establishes the link between the $SU(1, 1)$ coherent states and fractal-like self-similarity [1,9,10].

Our result:

- the thermal condensate, which turns out to behave as a perfect fluid giving a contribution to the radiation of the universe, reproduces the behavior of CMB and CNB.

- the vacuum condensate of the CMB and CNB is a coherent state and exhibits fractal self-similarity properties.

Conclusion. The thermal vacuum structure at the temperature of the CMB has been studied in the framework of Thermo-Field Dynamics. The expectation value on the thermal vacuum of the energy momentum tensor density of photons agrees with the energy density and pressure of the CMB. The estimated lower bound of the sum of the active neutrino masses has been found to be consistent with thermal states at the CNB temperature. The thermal vacuum has fractal structure. For a more detailed discussion and the mathematical formalism see [1].

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